The Information Dynamics of Phase Transitions in Random Boolean Networks

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Abstract

Random Boolean Networks (RBNs) are discrete dynamical systems which have been used to model Gene Regulatory Networks. We investigate the well-known phase transition between ordered and chaotic behavior in RBNs from the perspective of the *distributed computation* conducted by their nodes. We use a recently published framework to characterize the distributed computation in terms of its underlying *information dynamics*: information *storage*, information *transfer* and information *modification*. We find maximizations in information storage and coherent information transfer on either side of the critical point, allowing us to explain the phase transition in RBNs in terms of the intrinsic distributed computations they are undertaking.

Introduction

The information dynamics of distributed computation has recently emerged as an important tool for studying complex systems, e.g. information transfer in cellular automata (Lizier et al., 2008b, 2007). We believe that information dynamics are particularly relevant to networked systems: while network's structure has attracted much attention (Aldana, 2003), their time-series dynamics are "much less well understood" (Mitchell, 2006). Although the time-series dynamics of state-space trajectories and damage spreading are established, Mitchell (2006) suggests that "the main challenge is understanding the dynamics of the propagation of information ... in networks, and how these networks process such information."

Several studies have investigated the propagation and the processing of information in networks, in particular reporting phase transitions of these properties between ordered and chaotic regimes. Solé and Valverde (2001) investigated the effect of varying the message generation rate in a model of computer networks, finding phase transitions maximizing the number of packets actually delivered and the mutual information in the status of random node pairs. They infer that information transfer is maximized at the critical state. Kinouchi and Copelli (2006) investigated varying the "branching ratio" (effectively an activity level) in a network of excitable elements, finding phase transitions maximizing the dynamic range of the element's output, and inferring a maximization of information processing at criticality.

We are particularly interested in investigating the information dynamics of Random Boolean Networks (RBNs) (Kauffman, 1993), in part because of the power in their generality as discrete dynamical network models with a large sample space available. Also, they have a well-known phase transition from ordered to chaotic dynamics, in terms of length of transients in phase space with respect to average connectivity or activity level. We are also motivated by their popularity as models of Gene Regulatory Networks (GRNs). Perhaps most importantly, there have been several recent attempts to study the computational properties of RBNs (in particular information transfer). Here, Ribeiro et al. (2008) measure mutual information in the states of random node pairs as a function of connectivity in the network, and Rämö et al. (2007) measure the uncertainty (entropy) in the size of perturbation avalanches as a function of an order parameter. Both find maximization near the critical point, claiming that their results imply maximization of information propagation in this regime.

While these results are interesting, they do not directly measure the information dynamics claimed, e.g. none of the purported measures of information transfer properly measure directed, dynamic flows of information. Measures of model or task specific properties (by Solé and Valverde (2001), Kinouchi and Copelli (2006) and Rämö et al. (2007)) are qualitatively appealing but give no insights into the underlying quantitative nature of the information dynamics, while mutual information between random pairs of nodes (by Ribeiro et al. (2008) and Solé and Valverde (2001)) measures dynamic correlation across the collective which may result from an information transfer but is not a measure of it. (A more generic measure of "information transfer" in networks is presented in (Solé and Valverde, 2004), however it is a static measure of structure rather than a directed, dynamic flow of information.)

In this paper, we examine the information dynamics of RBNs from the perspective of the distributed computation undertaken by the nodes of the network in computing their attractor. We apply a recently published framework to characterize the information dynamics of the distributed computation in terms of the elements of Turing universal computation: information storage, information transfer and information modification (Lizier et al., 2007). Our perspective of computation in RBNs is an important one, underlined by the comments of Mitchell (2006) on information dynamics in networks, and by the general importance attributed to information processing in biological systems (Polani et al., 2007; Gershenson, 2004a). Importantly, the perspective of distributed computation is unique in quantitatively aligning with our understanding of information storage, transfer and modification (Lizier et al., 2007).

We begin with overviews of RBNs and our framework for the information dynamics of distributed computation, and subsequently discuss how the framework will be applied to RBNs. We then present the results of this application, demonstrating that information storage and transfer are both maximized in the vicinity of the phase transition between ordered and chaotic dynamics. Importantly, we demonstrate a shift from the dynamics being dominated by information storage in the ordered regime, to a balance of information storage and transfer around the critical point, and a further shift to the dominance of information transfer (in particular higher order interactions) in the chaotic regime. Near the critical point we observe maximum capability for coherent computation, with relatively few but high-impact non-trivial information modification events. It is likely that these insights on the nature of computation in the vicinity of order-chaos phase transitions will be applicable to other complex systems.

Random Boolean Networks

Random Boolean Networks are a class of generic discrete dynamical network models. They are particularly important in artificial life, since they were proposed as models of gene regulatory networks by Kauffman (1993). See also Gershenson (2004a) for another thorough introduction to RBNs.

An RBN consists of N nodes in a directed *network* structure. The nodes take *boolean* state values, and update their state values in time as a function of the state values of the nodes from which it has incoming links. The network topology (i.e. the adjacency matrix) is determined at *random*, subject to whether the in-degree for each node is constant or stochastically determined given an average in-degree $\overline{K}$ (giving a Poissonian distribution). It is also possible to bias the network structure, e.g. toward scale-free degree distribution (Aldana, 2003). Given the topology, the deterministic boolean function or lookup table by which each node computes its next state from its neighbors is also decided at *random* for each node, subject to a probability p of producing "1" outputs (p close to 1 or 0 gives low activity, close to 0.5 gives high activity). The nodes here are heterogeneous agents: there is no spatial pattern to the network structure (indeed there is no inherent concept of locality), nor do the nodes have the same update functions. (Though, of course either of these can arise at random). Importantly, the network structure and update functions for each node are held static in time ("quenched"). In classical RBNs (CRBNs), the nodes all update their states synchronously.[1]

The synchronous nature of CRBNs, their boolean states and deterministic update functions give rise to a global state space for the network as a whole with deterministic transient trajectories ultimately leading to either fixed or periodic attractors in finite-sized networks (Wuensche, 1997). Effectively, the transient is the period in which the network is *computing* its steady state attractor.

RBNs are known to exhibit three distinct phases of dynamics, depending on their parameters: ordered, chaotic and critical. At relatively low connectivity (i.e. low degree K) or activity (i.e. p close to 0 or 1), the network is in an ordered phase, characterized by high stability of states and strong convergence of similar macro states in state space. Alternatively, at relatively high connectivity and activity, the network is in a chaotic phase, characterized by low stability of states and divergence of similar macro states. In the critical phase (the *edge of chaos* (Langton, 1990)), there is percolation in nodes remaining static or updating their values, and uncertainty in the convergence or divergence of similar macro states. This phase transition is typically quantified using a measure of sensitivity to initial conditions, or damage spreading. Following Gershenson (2004c), we take a random initial state A of the network, invert the value of a single node to produce state B, then run both A and B for many time steps (enough to reach an attractor is most appropriate). We then use the Hamming distance:

$$D(A,B) = \frac{1}{N}\sum_{i=1}^{N} |a_i - b_i|, \tag{1}$$

between A and B at their initial and final states to obtain a convergence/divergence parameter δ:

$$\delta = D(A,B)_{t\to\infty} - D(A,B)_{t=0}. \tag{2}$$

(Note $D(A,B)_{t=0} = 1/N$). Finding $\delta < 0$, implies the convergence of similar initial states, while $\delta > 0$ implies their divergence. For fixed p, the critical value of $\overline{K}$ between the ordered and chaotic phases is (Derrida and Pomeau, 1986):

$$K_c = \frac{1}{2p(1-p)}. \tag{3}$$

[1] There has been some debate about the best updating scheme to model GRNs (Darabos et al., 2007), and variations on the synchronous CRBN model are known to produce different behaviors. However, the relevant phase transitions are known to exist in all updating schemes, and their properties depend more on the network size than on the updating scheme (Gershenson, 2004b). As such, the use of CRBNs is justified for ensemble studies such as ours (Gershenson, 2004c).

For $p = 0.5$, we have $K_c = 2.0$. The standard deviation of δ peaks slightly inside the chaotic regime for finite-sized networks, indicating the widest diversity of networks for those parameters (Gershenson, 2004b).

Much has been speculated on the possibility that gene regulatory and other biological networks function in (or evolve to) the critical regime (see Gershenson (2004a)). It has been suggested that computation occurs more naturally with the balance of order and chaos there (Langton, 1990), possibly with information storage, propagation and processing capabilities maximized (Kauffman, 1993). Here we seek to improve on previous attempts to measure these computational properties, with a thorough quantitative study of the information dynamics in RBNs.

Information dynamics

Information theory (MacKay, 2003) is the natural domain to look for a framework to describe the information dynamics in complex systems, and indeed information theory is proving to be a useful framework for the analysis and design of complex systems, e.g. (Klyubin et al., 2004). The fundamental quantity is the (Shannon) *entropy*, which represents the uncertainty in a sample x of a random variable X: $H_X = -\sum_x p(x) \log_2 p(x)$ (all with units in bits). The *joint entropy* of two random variables X and Y is a generalization to quantify the uncertainty of their joint distribution: $H_{X,Y} = -\sum_{x,y} p(x,y) \log_2 p(x,y)$. The *conditional entropy* of X given Y is the average uncertainty that remains about x when y is known: $H_{X|Y} = -\sum_{x,y} p(x,y) \log_2 p(x|y)$. The *mutual information* between X and Y measures the average reduction in uncertainty about x that results from learning the value of y, or vice versa: $I_{X;Y} = H_X - H_{X|Y}$. The *conditional mutual information* between X and Y given Z is the mutual information between X and Y when Z is known: $I_{X;Y|Z} = H_{X|Z} - H_{X|Y,Z}$. Finally, the *entropy rate* is the limiting value of the entropy of the next state x of X conditioned on the previous $k-1$ states $x^{(k-1)}$ of X: $H_{\mu X} = \lim_{k\to\infty} H\left[x|x^{(k-1)}\right] = \lim_{k\to\infty} H_{\mu X}(k)$.

We have previously proposed a framework for the local information dynamics of distributed computation in (Lizier et al., 2007). The framework describes computation in terms of information storage, transfer and modification at each spatiotemporal point in a complex system.

The *information storage* of an agent in the system is the amount of information in its past that is relevant to predicting its future. The *excess entropy* is the total information stored by the agent (Feldman and Crutchfield, 2003), while the *active information storage* is the stored information that is currently in use in computing the next state of the agent (Lizier et al., 2007). We focus on the active information since it yields an immediate contrast in the relative contributions of storage and transfer to each computation. As shown in Fig. 1, the *local active information storage* for agent X is defined as the local (or unaveraged) mutual information

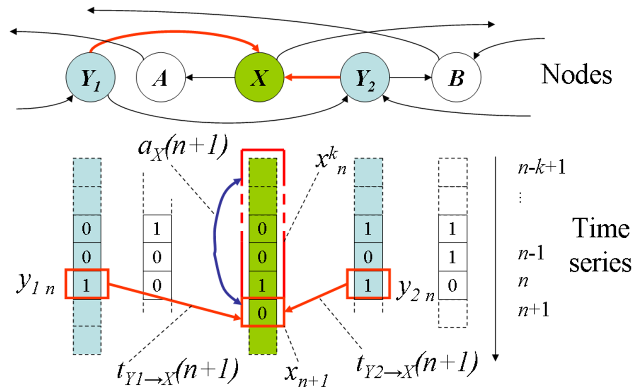

Figure 1: Information dynamics in a distributed network. For node X, this figure displays the local active information $a_X(n+1,k)$ and the local transfer entropies $t_{Y_1\to X}(n+1)$ and $t_{Y_2\to X}(n+1)$ from each of the causal information sources $V_X \in \{Y_1, Y_2\}$ at time $n+1$.

between its semi-infinite past $x_n^{(k)}$ (as $k \to \infty$) and its next state x_{n+1} at time step $n+1$:

$$a_X(n+1) = \lim_{k\to\infty} \log_2 \frac{p(x_n^{(k)}, x_{n+1})}{p(x_n^{(k)})p(x_{n+1})}, \tag{4}$$

with $a_X(n,k)$ representing an approximation with finite history length k. The *active information* is the average over time (or equivalently weighted by the distribution of $(x_n^{(k)}, x_{n+1})$): $A_X(k) = \langle a_X(n,k)\rangle$. From our computational perspective, an agent can store information regardless of whether it is causally connected with itself; i.e. for RBNs, this means whether or not the node has a self-link. This is because information storage can be facilitated in a distributed fashion via one's neighbors, which amounts to the use of stigmergy (e.g. see Klyubin et al. (2004)) to communicate with oneself (Lizier et al., 2008a). Finally, the local entropy for any agent is the sum of the local active information and the local entropy rate $h_{\mu X}(n,k)$ (for any k):

$$h_X(n) = a_X(n,k) + h_{\mu X}(n,k), \tag{5}$$

with their averages also related in this way. In a deterministic system, the entropy rate represents the joint contribution from the causal information sources to the destination (Lizier et al., 2008a), though it does not specify the information transfered from any particular one of those sources.

The *information transfer* between a source and a destination agent is defined as the information provided by the source about the destination's next state that was not contained in the past of the destination. The information transfer is formulated in the *transfer entropy*, introduced by Schreiber (2000) to address concerns that the mutual information (as a de facto measure of information transfer) was a symmetric measure of statically shared information. The *local transfer entropy* (Lizier et al., 2008b) from a source agent Y to a destination agent X is the local mutual information between the previous state of the source[2] y_n and the next state of the destination x_{n+1}, *conditioned* on the semi-infinite past of the destination $x_n^{(k)}$ (as $k \to \infty$):

$$t_{Y \to X}(n+1) = \lim_{k \to \infty} \log_2 \frac{p(x_{n+1}|x_n^{(k)}, y_n)}{p(x_{n+1}|x_n^{(k)})}. \quad (6)$$

Again, $t_{Y \to X}(n,k)$ represents finite-k approximation, and the *transfer entropy* is the (time or distribution) average: $T_{Y \to X}(k) = \langle t_{Y \to X}(n,k) \rangle$. The local transfer entropy is shown in Fig. 1. The transfer entropy can also be formulated to condition on the states $v_{x,y,n}$ of all *causal information contributors* to the destination (the set V_X) except the source Y, so as to completely account for the contribution of Y. This formulation is known as the *complete* transfer entropy (Lizier et al., 2008b), with average and local values defined as:

$$T^c_{Y \to X}(n+1) = \langle t^c_{Y \to X}(n+1) \rangle , \quad (7)$$

$$t^c_{Y \to X}(n+1) = \lim_{k \to \infty} \log_2 \frac{p(x_{n+1}|x_n^{(k)}, y_n, v_{x,y,n})}{p(x_{n+1}|x_n^{(k)}, v_{x,y,n})}, \quad (8)$$

$$v_{x,y,n} = \{z_n | \forall Z \in V, Z \neq Y\} . \quad (9)$$

The formulation in Eq. (6) is then labeled the *apparent* transfer entropy. Importantly, the transfer entropy properly measures a directed, dynamic flow of information, unlike mutual information measures used by Ribeiro et al. (2008) and Solé and Valverde (2001) which measure correlations only.

Information modification has been described as interactions between transmitted and/or stored information which result in a modification of one or the other (Langton, 1990). In (Lizier et al., 2007), we observed that *negative* values of $a_X(n)$ and $t_{Y \to X}(n)$ indicated misinformation or surprise regarding a given local outcome. We hypothesized that the sum of the local active information storage and apparent transfer entropy from each causal information contributor would be *negative* in a local information modification event, where no information source contained enough predictive power to overcome the misinformation generated by the other sources in the information "collision". This sum is known as the local *separable information*:

$$s_X(n) = a_X(n) + \sum_{Y \in V, Y \neq X} t_{Y \to X}(n). \quad (10)$$

Again, $s_X(n,k)$ represents finite-k approximation, and the *separable information* is the average $S_X(k) = \langle s_X(n,k) \rangle$.

[2]The transfer entropy can be formulated using the l previous states of the source. However, where only the previous state is a causal information contributor (as for RBNs), it is sensible to set $l = 1$ to measure direct transfer only at step n.

In Fig. 1, we have $s_X(n,k) = a_X(n,k) + t_{Y_1 \to X}(n,k) + t_{Y_2 \to X}(n,k)$. Positive local values of $s_X(n,k)$ indicate trivial information modification events, while negative local values of $s_X(n,k)$ indicate non-trivial information modifications events where the information sources interact in a non-trivial manner.

This framework was applied to cellular automata (CAs), which are effectively an ordered lattice-style sub-class of RBNs (Wuensche, 1997), in (Lizier et al., 2007). The framework quantified blinkers and regular domains as the dominant information storage elements, particles (gliders and domain walls) as the dominant information transfer agents, and particle collisions as the dominant (non-trivial) information modification events. These results align with existing conjecture on the nature of distributed computation in CAs, providing significant impetus for the use of this framework to analyze computation in other complex systems.

Information dynamics of RBNs

In this study, we seek to measure the *average* information dynamics of RBNs as a function of average in-degree or connectivity $\overline{K}$. For the RBNs simulated here, we use $N = 250$, Poissonian distributed in-degree for each node based on average in-degree $\overline{K}$, $p = 0.5$ (no bias in rules), and CRBNs with synchronous updating. Also, we do not bias the network structure, allowing comparison with the majority of existing RBN publications. The RBNs are modeled using enhancements to Gershenson's RBNLab software (http://rbn.sourceforge.net).

We measure the average entropy, entropy rate, and active information for each node in a given RBN (e.g. $A_X(k)$), then average these over each node in the RBN (to get e.g. $\langle A_X(k) \rangle$), then average these network averages over many networks generated for each $\overline{K}$ (at least 250) to determine the average values as a function of $\overline{K}$ (denoting this, e.g., as $A_X(k, \overline{K})$). Similarly, the average apparent and complete transfer entropies are measured for (at least 50) sample pairs of causally linked nodes (unlike the mutual information measurements by Ribeiro et al. (2008) and Solé and Valverde (2001) for *random* node pairs), averaged once to obtain network averages, and again over many networks to obtain averages as a function of $\overline{K}$.

While the *local* information dynamics are known to provide significantly greater insights into the distributed computation than their averaged counterparts (Lizier et al., 2007, 2008b), the averages will provide sufficient summaries regarding the ensemble properties with respect to $\overline{K}$. A hybrid approach is taken for the separable information; the average $S(k, \overline{K})$ is computed in a similar manner to the other metrics, however we also record the balance between its positive and negative local values (trivial and non-trivial information modifications respectively) $S^+_X(k, \overline{K})$ and $S^-_X(k, \overline{K})$ in contributing to the average. For a given node, we have for example $S^+_X(k) = \langle s^+_X(n,k) \rangle$, where:

$$s^+_X(n,k) = \begin{cases} s_X(n,k) & \text{if } s_X(n,k) \geq 0 \\ 0 & \text{if } s_X(n,k) < 0 \end{cases}. \quad (11)$$

We seek to approximate an infinitely-sized network, and so avoid running the RBN for too many time steps because the computation is completed once the network reaches a periodic or fixed attractor (inevitable for finite-sized RBNs). For each simulation from an initial randomized state, we ignore a short initial transient of 30 steps to allow the network to settle into the main phase of the computation, then allow evolution over 400 time steps. Importantly, since the nodes in each RBN are heterogeneous agents, the probability distribution functions for each measure must be computed for each node individually rather than combining observations across all nodes (as could be done for the homogeneous agents in CAs (Lizier et al., 2007)). In order to properly sample the dynamics of each node in each RBN and generate enough data for the information theoretic calculations, many repeat runs from random initial states are required for each network (at least 4480 are used). For these calculations, one should use as large a history length k as facilitated by the number of observations (Lizier et al., 2008b); here we find $k \approx 13$ provides reasonable convergence for a reasonable number of repeat runs.

It has been hypothesized that RBNs close to the critical state possess a maximal information transfer capability (e.g. (Rämö et al., 2007)), which is generalized in the "edge of chaos" hypothesis (Langton, 1990): that systems exhibiting critical dynamics in the vicinity of a phase transition maximize their computational properties (see Kauffman (1993) regarding RBNs in particular). More specifically, Langton (1990) suggests that an intermediate level of information transfer and storage gives rise to complex computation in critical dynamics, with too much of either decaying the computational capability. This is at odds with suggestions of the maximization of information transfer in this regime, e.g. (Rämö et al., 2007; Solé and Valverde, 2001).

Our experiments aim to provide insight here. It is simple to foresee the average active information and apparent transfer entropy being zero in the extreme ordered regime (with fast freezing at point attractors) and in the extreme chaotic regime (where the high level of interactions overwhelm information storage and obscure the apparent contribution of each information source). It seems reasonable that both would be maximized, on average, in the interim near the critical region, where the dynamics support long correlations across space and time. On the other hand, we predict that the complete transfer entropy (which has been suggested to reveal higher information contributions as the level of interactions increases (Lizier et al., 2008a)) will continue to increase with the connectivity into the chaotic regime. Indeed, we observed that relatively high values of the apparent transfer entropy indicated the capacity for coherent local information transfer structures (i.e. gliders in CAs). We hypothesized there that an increasing of the complete transfer entropy in the chaotic regime indicated a higher level of interactions in conjunction with the loss of this coherence.

Important caveats are provided by criticisms of the edge of chaos hypothesis, e.g. see Mitchell et al. (1993). In examining *average* computational properties as a function of RBN parameters, we emphasize that there is in general a very large range of network realizations and consequently of behaviors possible for each parameter set. The local information dynamics of computation will provide much more detailed insights *for a given* RBN (as for CAs in (Lizier et al., 2008b)) than averages over nodes, networks and network sets discussed here. That being said, these averages can provide important insights into the computational properties as a function of RBN parameters, so long as we remember that the average results are akin to likelihoods rather than certainties, albeit likelihoods that are much stronger in the limit of infinite system size.

Results and discussion

Fig. 2 shows that the the average single node entropy $H_X(\overline{K})$ simply increases as a function of $\overline{K}$, as expected since the level of activity in the network is increasing with this parameter. More importantly, Fig. 2 also plots the average active information $A_X(k = 14, \overline{K})$ and entropy rate $H_{\mu X}(k = 14, \overline{K})$, showing that the active information rises then reaches a maximum near to the critical phase ($\overline{K} = 2$) before falling away, while the entropy rate only begins to rise near the critical phase then continues to rise and approach the entropy in the chaotic phase. Since the entropy is the sum of the active information and entropy rate (Eq. (5)), we can now begin to describe the phase transition in terms of computation: the ordered phase is dominated by information storage (information contained in the past of the node about its next state), the chaotic phase is dominated by information transfer (information from incoming links about the next state which was not contained in the node's past), while there appears to be something of a balance between the two near the critical phase.

We then examine the constituency of the information contributed from incoming links, the total of which is the entropy rate. Fig. 2 also plots the average *apparent* transfer entropy $T_{Y \to X}(k = 14, \overline{K})$ for each link, demonstrating that this quantity too rises to a maximum value close to the critical phase, then falls away. In contrast, Fig. 2 additionally plots the average *complete* transfer entropy $T^c_{Y \to X}(k = 13, \overline{K})$ for each link, which also begins to rise close to the critical phase but continues to increase into the chaotic phase. We see therefore that in the first stage of the shift toward the dominance of information transfer, the sources can be observed to have a significant influence on the destination (in the context of the destination's history) without considering the effect of the other causal sources (i.e.

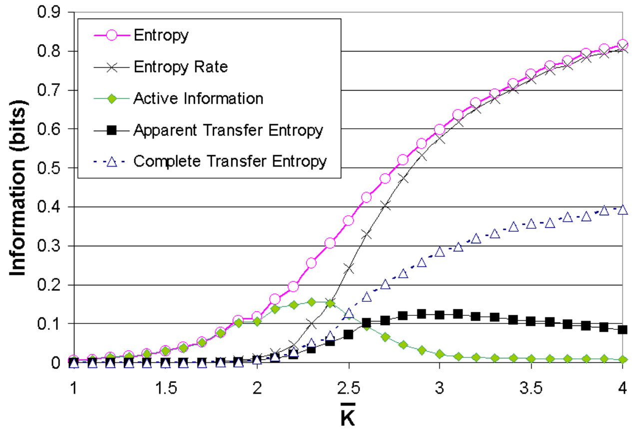

Figure 2: Average information dynamics versus average connectivity $\overline{K}$ for networks of size $N = 250$. Plotted here are the average entropy $H_X(\overline{K})$, entropy rate $H_{\mu X}(k = 14, \overline{K})$, active information $A_X(k = 14, \overline{K})$, apparent transfer entropy $T_{Y\rightarrow X}(k = 14, \overline{K})$ and complete transfer entropy $T^c_{Y\rightarrow X}(k = 13, \overline{K})$. The information required to predict the next state of each node is dominated by information storage at low $\overline{K}$ and by information transfer at higher $\overline{K}$ (first by coherent then interaction effects). Error bars (omitted) are on the scale of the data points for all plots.

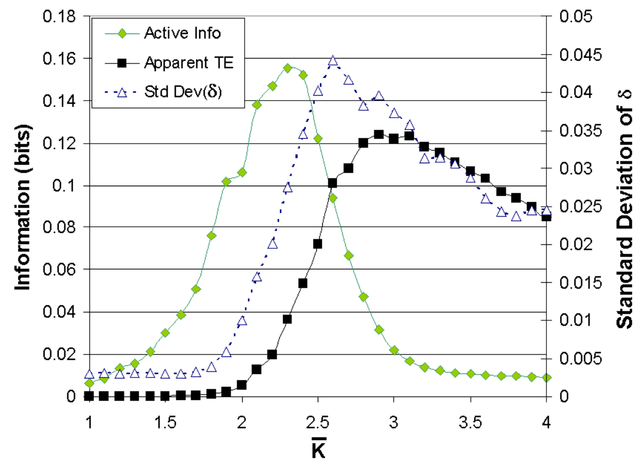

Figure 3: Maximizations in active information $A_X(k = 14, \overline{K})$ and apparent transfer entropy $T_{Y\rightarrow X}(k = 14, \overline{K})$ as a function of average connectivity $\overline{K}$ for $N = 250$, shown with respect to the standard deviation of the convergence/divergence parameter δ. This indicates that information storage peaks just on the ordered side of the phase transition, while (coherent) information transfer peaks just on the chaotic side of the phase transition.

$T_{Y\rightarrow X}(k = 14, \overline{K})$ is relatively high). In this regime, there is greater potential for *coherent* information transfer structures to propagate. However, as the activity level in the RBNs continues to rise with the average connectivity $\overline{K}$, the apparent effect of each source is swamped by the activity of the other causal sources, leading $T_{Y\rightarrow X}(k = 14, \overline{K})$ to fall away. Considering also the increase in $T^c_{Y\rightarrow X}(k = 13, \overline{K})$ (which does account for the other sources), we see that the level of interaction is increasing with the connectivity of the network. In the chaotic regime, the influence of any one information source can only be properly identified by taking all of the other sources into account also. These complimentary measures of information transfer provide different but useful insights, and give impetus to our hypothesis in (Lizier et al., 2008a) regarding the relative values of the apparent and complete components of information transfer in order-chaos phase transitions.

Next, we compare these maximizations to the phase transition as measured using the standard deviation of the convergence/divergence parameter δ (from Eq. (2)).[3] In Fig. 3 we see that the information storage peaks slightly within the ordered phase from the critical region, while the information transfer peaks slightly within the chaotic phase. Importantly, it is the apparent transfer entropy that peaks here (indicating the capability for coherent information transfer), as distinct from the complete transfer entropy which continues to increase into the chaotic phase. As per footnote 3, we expect the relative positions of these maximizations to be maintained around the critical phase as $N \rightarrow \infty$, with both likely to become closer to the critical point in this limit (as for the measure of correlation by Ribeiro et al. (2008)). The relative positions of the maximizations are quite interesting, because they align with existing conjecture on the nature of computation around phase transitions which typically associates information storage with the ordered phase and information transfer with the chaotic phase (e.g. (Langton, 1990)). Both the information storage and transfer appear to be driving the dynamics toward the critical phase, but from different sides of the phase transition.

We can also add quantitative evidence to the conflicting conjecture around whether information transfer is found at an intermediate (Langton, 1990) or maximum level (Solé and Valverde, 2001) at criticality. For RBNs, it is maximized close to criticality where one measures the apparent influence of a source in isolation, but equally it is at an intermediate level where the measurement considers the other

[3] δ was confirmed to change sign close to $\overline{K} = 2$ here (as per (Gershenson, 2004b)), with a subsequent slow increase after $\overline{K} = 2$ (known to be a finite-N effect). The standard deviation of δ is maximized during this increase in the chaotic regime (Gershenson, 2004b). Certain other measures suggested to indicate the critical phase are known to be shifted into the chaotic regime for finite-N, e.g. (Ribeiro et al., 2008). Given impetus as an indicator of the critical phase by the related measure of Rämö et al. (2007), we use the standard deviation of δ as guide to the relative regions of dynamics in finite-N networks.

causal information sources also. If these findings apply to such phase transitions in general, then both sources of conjecture appear to be well-founded, being resolved in these two different methods of measuring information transfer.

Indeed, we previously conjectured the capacity for the coherence of information transfer (provided by relatively large apparent transfer entropy) to be an important feature of complex dynamics in (Lizier et al., 2008a). Further insight into the coherent nature of the computation in the RBN is provided by the separable information $S_X(k, \overline{K})$. Fig. 4 shows that $S_X(k, \overline{K})$ is maximized for approximately the same values of $\overline{K}$ as the apparent transfer entropy (though it is slightly more spread out). This can be explained with reference to its positive and negative components, $S_X^+(k, \overline{K})$ and $S_X^-(k, \overline{K})$. We see from Fig. 4 that the early rise in the separable information is driven by $S_X^+(k, \overline{K})$ (trivial information modifications), with a peak occurring before $S_X^-(k, \overline{K})$ (non-trivial information modification events) rises and consequently reduce the total. As the connectivity $\overline{K}$ is further increased, $S_X^+(k, \overline{K})$ begins to fall whereas $S_X^-(k, \overline{K})$ continues to rise. Near the critical phase, at the peak of the separable information, note that there is in fact a relatively low incidence of non-trivial information modification events (i.e. $S_X^-(k, \overline{K})$ is low). This is interesting because of the importance placed on these events in computation, e.g. they are manifested as particle collisions in CAs. It appears that if the amount of non-trivial information modification events or information collisions is too large, the capacity of the system for complex computation is reduced. It is likely that this is due to a large amount of collisions eroding the coherent nature of the information storage and transfer within the system, disturbing the computation and reducing their own impact. A maximization of separable information, should perhaps be interpreted as maximizing the bandwidth for coherent information storage and modification, while allowing a smaller number of *high-impact* non-trivial information modification events in the coherent computation.

Finally, we note that all of the information dynamics described here experience maximum standard deviation in the vicinity of the critical region (not shown). This indicates maximal diversity in the information dynamics throughout the RBNs in this regime, as observed for other measures (e.g. (Gershenson, 2004b)).

Conclusion

We have described results which quantify the fundamental nature of computation around the critical phase in RBNs. The dynamics of RBNs are dominated by information storage in the ordered phase, with the level of information storage increasing with connectivity in the network. The increasing connectivity facilitates increasing activity, giving rise to an increasing level of information transfered from linked nodes. These two operations of universal computation appear to be in balance around the critical point. After

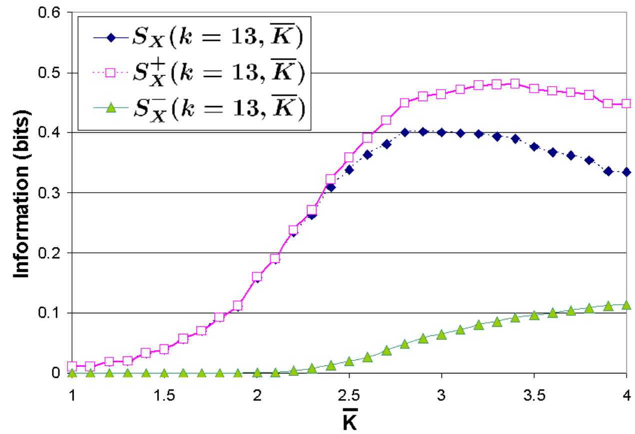

Figure 4: Separable information $S_X(k = 13, \overline{K})$ and its positive and negative components, $S_X^+(k = 13, \overline{K})$ and $S_X^-(k = 13, \overline{K})$ respectively, versus average connectivity $\overline{K}$ for $N = 250$. Trivial information modification (high $S_X^+(k)$) dominates the dynamics at low $\overline{K}$, while the amount of non-trivial information modification rises with $\overline{K}$.

this, information transfer continues to increase with connectivity, reducing the capacity for information storage. Near the critical point, there is a large amount of trivial information modifications, providing the capability for coherent information transfer and storage to flourish and indeed maximize, and allowing the small number of non-trivial information modifications to have a large impact on the coherent computation. As connectivity continues to increase, the information transferred from any single node observed in isolation initially appears strong, peaking slightly into the chaotic regime. With further increases however, the interaction between the nodes begins to dominate and erodes the capacity for coherent computation.

This new understanding of the information dynamics in RBNs near the critical phase is important, because there is evidence that the gene regulatory networks they model operate in this critical regime (Rämö et al., 2006). The implication here is that GRNs have evolved to a form facilitating maximum coherent computational capability. Furthermore, this study of RBNs represents the first exploration of an order-chaos phase transition using this framework for information dynamics: the results here are likely to be pertinent to order-chaos phase transitions in other systems.

We intend to continue our investigation of the information dynamics in RBNs, e.g. the effect of varying network size. Given the fundamental nature of the computational properties here, we expect to be able to describe the manner in which these information dynamics underpin other measures of the phase transition in RBNs, e.g. high interactivity (measured by complete transfer entropy and negative component of separable information) leads to large perturbation avalanche sizes. We expect that the choice of RBN updating scheme will have little effect on the fundamentals of the phase transitions reported here, though this should be investigated. Furthermore, we intend to explore the effect of different topologies, in particular scale-free topologies (since most biological networks are scale-free with an exponent putting them near the critical point (Aldana, 2003)). Finally, we intend to investigate whether the information dynamics here can be used to drive evolution or self-tuning adaptation of RBNs to produce critical networks. Such an experiment could provide evidence that an underlying capacity for computation may have been a driver in GRN evolution.

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